Logical Reasoning

Lesson Synopsis:

In this lesson, students will explore logic statements and determine the conditions that make them TRUE or FALSE. Students will explore conditionals and their related statements in both a real world and mathematical setting to help students develop an understanding of logic and the role it plays in geometry. Deductive reasoning and inductive reasoning will be introduced and contrasted. Each will be examined from a real world perspective and transitioned into their applications in a mathematical setting. The concept of a proof will be introduced for the first time.

TEKS:

G.1	***Geometric structure. The student understands the structure of, and relationships within, an axiomatic system.***
G.1A	**Develop an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems.**
G.3	***Geometric structure. The student applies logical reasoning to justify and prove mathematical statements.***
G.3A	**Determine the validity of a conditional statement, its converse, inverse, and contrapositive.**
G.3B	**Construct and justify statements about geometric figures and their properties.**
G.3C	**Use logical reasoning to prove statements are true and find counter examples to disprove statements that are false.**
G.3D	**Use inductive reasoning to formulate a conjecture.**
G.3E	**Use deductive reasoning to prove a statement.**

GETTING READY FOR INSTRUCTION

Performance Indicator(s):

- Develop an awareness of the application of TRUE and FALSE statements to make connections and justify conjectures. Provide examples, counterexamples, and use Venn Diagrams to justify why the statement is TRUE or FALSE. (G.1A; G.3C)
  ELPS ELPS: 1E, 2E, 2I, 3H, 4D, 5G
- Construct a conditional statement and its converse, inverse, and contrapositive. Determine and justify the validity of statement. (G.1A; G.3A, G.3B, G.3C)
  ELPS ELPS: 1E, 2E, 2I, 3H, 4D, 5G
- Apply inductive reasoning to formulate a conjecture supported by specific examples that illustrates a theorem. Apply deductive reasoning to prove a statement or theorem. (G.1A; G.3B, G.3C, G.3D, G.3E)
  ELPS ELPS: 1E, 2E, 2I, 3J, 4K, 5G

KEY Understandings and Guiding Questions:

- Logic statements such as AND and OR can be used to examine problem situations and determine truth values.
  — How do the truth values of a statements guide our mathematical reasoning?
  — What conditions make a statement TRUE?
  — What conditions make a statement FALSE?
  — How are counterexamples used to disprove a statement?
  — How are quantifiers used to change the truth value of a statement?
  — What conditions make an AND statement TRUE?
  — What conditions make an AND statement FALSE?
  — What conditions make an OR statement TRUE?
  — What conditions make an OR statement FALSE?

- A conditional statement has three related statements, the converse, inverse and contrapositive, which are useful in constructing arguments and examining problem situations.
  — What types of statements in geometry are typically stated as conditionals?
  — What part do conditionals play in geometric structure?
  — How do you determine if a conditional is TRUE?
  — How do you determine if a conditional is FALSE?
  — What are the components of a conditional?
  — What is the hypothesis of a conditional?
  — What is the conclusion of a conditional?
  — What are different ways that conditionals may be written or stated?
  — What are the related statements for a conditional?
  — What is the relationship between a conditional and its converse?
  — What is the relationship between a conditional and its inverse?
  — What is the relationship between a conditional and its contrapositive?
  — Which related statement is always equivalent to the original conditional?
- Inductive and deductive reasoning are processes to reach and justify conclusions.
  — What are two types of logical reasoning?
  — What is inductive reasoning?
  — What is deductive reasoning?
  — How can inductive reasoning be used to arrive at a conjecture or make an argument supporting a statement?
  — How can deductive reasoning be used to prove a statement?

Vocabulary of Instruction:

- counterexample
- AND statement
- OR statement
- conditional statement
- converse
- negate
- inverse
- contrapositive
- hypothesis
- conclusion
- inductive reasoning
- deductive reasoning

Materials:

- protractor
- ruler

Resources:

- Websites:
  — http://computer.howstuffworks.com/microprocessor2.htm
  — http://www.howstuffworks.com/boolean.htm
  — http://computer.howstuffworks.com/bytes1.htm
  — http://www.shodor.org/interactivate/activities/VennDiagrams/
  — http://en.wikipedia.org/wiki/Sherlock_Holmes
  — http://www.becca-online.org/images/Inductive_Reasoning_-_Benny.pdf
- **STATE RESOURCES:**
  — **Mathematics TEKS Toolkit:** Clarifying Activity/Lesson,/Assessments
  http://www.utdanacenter.org/mathtoolkit/index.php

  — **TEXTEAMS: Geometry for All Institute:** I – Functionally Speaking with Reason; Act. 1 (Logic Puzzle), Act. 2 (Riddles with Holes), Act. 3 (Inductive/Deductive); II – Transformationally Speaking with Reflection; Act. 5 (Is It…or Is It Not? Rubipink and Racroz)

  — **TEXTEAMS: High School Geometry: Supporting TEKS and TAKS:** I – Structure; 4.0 Conjectures, 4.1, Act. 1 (Triangles Tell It All)

Advance Preparation:

1. Transparency: **True/False and Logic** (1 per teacher)
2. Handout: **True/False Quiz** (1 per pair)

3. Handout: **True/False and Logic Statements** (1 per student)
4. Handout: **Venn There! Done That!** (1 per student)
5. Handout (optional): **Practicing True/False and Logic** (1 per student)
6. Handout: **On One Condition** (1 per student)
7. Handout: **Conditionals in Mathematics** (1 per student)
8. Transparency: **Modern Day Sherlock Holmes (pp. 1 & 2)** (1 per teacher)
9. Handout: **Modern Day Sherlock Holmes Questions** (1 per pair)
10. Handout: **Inductive and Deductive Reasoning** (1 per pair)
11. Handout: **Can You Justify It?** (1 per student)
12. Handout: **Looking at It Logically** (1 per student)

Background Information:

Students will most likely be unfamiliar with the reasoning process used to determine if a statement is *logically* TRUE or FALSE as they tend to think in generalities rather than specifics. In this lesson, students build their knowledge of logical reasoning. A working knowledge of TRUE/FALSE statements and counterexamples is essential for students to be able to work with conditionals and their related statements. Students use Venn diagrams to illustrate their understanding of logic statements. At this point in geometry, students have relied on inductive reasoning to formulate conjectures and create arguments. In particular students used inductive reasoning in a previous unit over *Patterns* to make predictions and formulate a rule based on a given pattern. Although students continue to use inductive reasoning throughout geometry, it is important for them to develop a deductive reasoning process essential for proof as well. Most importantly, students should realize that inductive reasoning leads to a conjecture which may or may not be true, while deductive reasoning always leads to a valid conclusion

GETTING READY FOR INSTRUCTION SUPPLEMENTAL PLANNING DOCUMENT

Instructors are encouraged to supplement, differentiate and substitute resources, materials, and activities to address the needs of learners. The Exemplar Lessons are one approach to teaching and reaching the Performance Indicators and Specificity in the Instructional Focus Document for this unit. A Microsoft Word template for this Planning document is located at www.cscope.us/sup_plan_temp.doc. If a supplement is created electronically, users are encouraged to upload the document to their Lesson Plans as a Lesson Plan Resource for future reference.

INSTRUCTIONAL PROCEDURES

Instructional Procedures

ENGAGE

1. Display the transparency: **True/False and Logic**. Use an appropriate reading/sharing strategy with your students to read the information and share it with a partner, group or the class as a whole.
2. After students have read and summarized the information the teacher may wish to pose the following facilitation questions.
   - **How do logic values (TRUE or FALSE) relate to the binary number system?** *Answers will vary. Sample: The binary system involves two choices off/on, 0/1, etc. True/false can be applied to the two choices.*
   - **What must occur to prove a statement true?** *All cases involving the statement must be true.*
   - **What must occur to prove a statement false?** *At least one case must be found to be false.*
3. Put students in pairs or small groups.
4. Distribute the handout: **True/False Quiz** to pair of students. Give students time to answer the questions.
5. Share out results in whole group discussion.

Notes for Teacher

NOTE: 1 Day = 50 minutes
Suggested Day 1 (1/4 day)
MATERIALS

- Transparency: **True/False and Logic** (1 per teacher)
- Handout: **True/False Quiz** (1 per pair)

TEACHER NOTE
The purpose of the Engage is to get students thinking about logical reasoning. Do not spend too long on this activity. The concepts will be further explored in the Explore/Explain 1 section of the lesson.

TEACHER NOTE
For further information regarding logic as applied to computing, links of interest are provided below.
http://computer.howstuffworks.com/microprocessor2.htm
http://www.howstuffworks.com/boolean.htm
http://computer.howstuffworks.com/byte

Instructional Procedures

Notes for Teacher

s1.htm

STATE RESOURCES
Mathematics TEKS Toolkit: Clarifying Activity/Lesson/Assessment may be used to reinforce these concepts or used as alternate activities.

TEXTEAMS: Geometry for All Institute: I – Functionally Speaking with Reason; Act. 1 (Logic Puzzle), Act. 2 (Riddles with Holes), Act. 3 (Inductive/Deductive); II – Transformationally Speaking with Reflection; Act. 5 (Is It…or Is It Not? Rubipink and Racroz) may be used to reinforce these concepts or used as alternate activities.

EXPLORE/EXPLAIN 1

1. Distribute the handout: **True/False and Logic Statements** to each student.
2. Go over the summary and examples 1 & 2 for "True/False" statements in whole-group instruction.
3. Have students work in pairs to complete the bottom of p. 1 and all of p. 2.
4. When students are finished, share results in whole group discussion.
5. Go over the summary for "AND/OR" statements in whole-group instruction.

Suggested Day 1 (1/2 day)

MATERIALS

- Handout: **True/False and Logic Statements** (1 per student)

TEACHER NOTE
In this activity, students will investigate the concepts that make statements true or false. Truth tables will also be constructed for "and" and "or" logic statements.

ELABORATE 1

1. Distribute the handout: **Venn There! Done That!** to each student.
2. Students should continue to work in pairs on the handout.
3. Students may complete the activity as homework if necessary.

Suggested Day 1 (1/4 day)

MATERIALS

- Handout: **Venn There! Done That!** (1 per student)

TEACHER NOTE
In this activity, students will extend their investigation of true/false and logic statements using the Venn diagram.

SUPPLEMENTAL MATERIALS

- Handout (optional): **Practicing True/False and Logic** (1 per student)

If some students need extra practice on the concepts of true/false and logic statements, the supplemental activity can be use.

TEACHER NOTE
For further information regarding Venn diagrams links of interest are provided below.

Instructional Procedures	Notes for Teacher
	http://www.shodor.org/interactivate/activities/VennDiagrams/

EXPLORE/EXPLAIN 2

1. Distribute the handout: **On One Condition** to each student.
2. Have students work individually to complete questions 1 & 2. Have students group in pairs or small groups to compare results.
3. Go over the summary and question 3 as an example in whole-group instruction.
4. Have students work in their pairs or small groups to complete the remainder of p. 2.
5. When students are finished, share results in whole-group discussion.
6. Go over the summary for conditional statements on p. 3 in whole-group instruction.

Suggested Day 2 (3/4 day)

MATERIALS

- Handout: **On One Condition** (1 per student)

TEACHER NOTE

In this activity, students will analyze a conditional statement to determine the hypothesis and conclusion. Students will also identify and analyze the converse, inverse, and contrapositive of an original conditional statement.

TEACHER NOTE

For more on truth tables and conditionals go to the following URL.
http://mcckc.edu/longview/CTAC/ttable.htm

TEACHER NOTE

Beginning on problem #7, students will be asked to negate the hypothesis and/or conclusion. This term may not be familiar to the students. You may need to explain and give an example.

— **Jane went to town.** (Original)
— **Jane did not go to town.** (Negated)

ELABORATE 2

1. Distribute the handout: **Conditionals in Mathematics** to each student.
2. Students should continue to work in pairs or small groups on the handout.
3. Students may complete the activity as homework if necessary.

Suggested Day 2 (1/4 day)

MATERIALS

- Handout: **Conditionals in Mathematics** (1 per student)

TEACHER NOTE

In this activity, students will extend their investigation of the concepts by investigating conditionals in mathematics.

EXPLORE/EXPLAIN 3

1. Display the transparency: **Modern Day Sherlock Holmes** and go over the first page in whole-group discussion. Set up the case scenario by having a student read the case.
2. Display the second page of the transparency. Put students in pairs or small groups and have them read and discuss the two profiles.
3. Distribute the handout: **Modern Day Sherlock Holmes Questions** to each pair or group.
4. Have students work in pairs or small groups to complete the questions about the two profiles. After a few minutes have groups share answers with the class.
5. Distribute the handout: **Inductive and Deductive Reasoning** to each student.
6. Go over the notes and examples in whole-group instruction.

Suggested Day 3 (3/4 day)

MATERIALS

- Transparency: **Modern Day Sherlock Holmes (pp. 1 & 2)** (1 per teacher)
- Handout: **Modern Day Sherlock Holmes Questions** (1 per pair)
- Handout: **Inductive and Deductive Reasoning** (1 per pair)

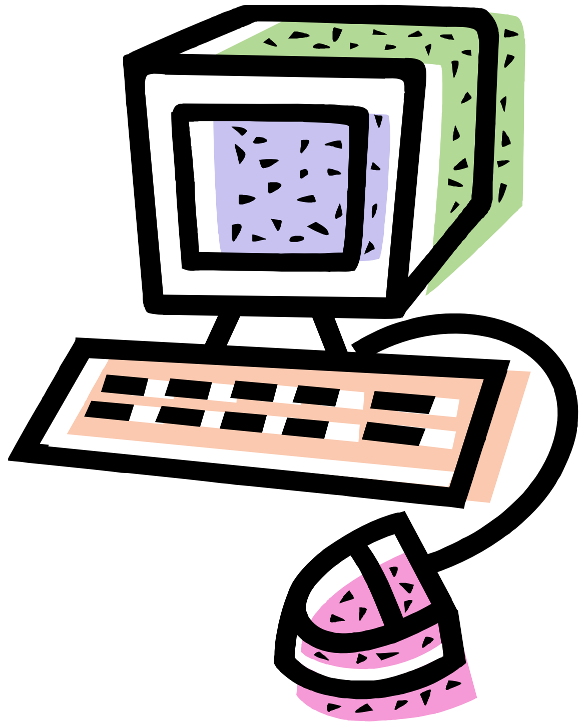
TEACHER NOTE

In this activity, students will investigate and compare inductive and deductive reasoning.

Instructional Procedures	Notes for Teacher
	MISCONCEPTION Most students falsely assume that inductive and deductive reasoning both provide a conclusive process for providing proof of a conjecture. However, the distinction is this: inductive reasoning provides an argument (evidence of past observations) for a conjecture to be formed but provides no proof. Deductive reasoning takes the process one step further and either proves or disproves a conjecture based on an axiomatic system. Although both types of reasoning are useful, inductive reasoning leads to a conjecture that is unproven and therefore may be FALSE, while deductive reasoning provides conclusive proof of a conjecture. **TEACHER NOTE** For further information regarding Sherlock Holmes links of interest are provided below. http://en.wikipedia.org/wiki/Sherlock_Holmes http://www.becca-online.org/images/Inductive_Reasoning_-_Benny.pdf
ELABORATE 3	**Suggested Day 3 (1/4 day)**
1. Distribute the handout: **Can You Justify It?** to each student. 2. Students should continue to work in pairs or small groups on the handout. 3. Students may complete the activity as homework if necessary.	MATERIALS • Handout: **Can You Justify It?** (1 per student) • protractor • ruler **TEACHER NOTE** In this activity, students will extend their knowledge of inductive and deductive reasoning to investigate proofs in geometry. **TEACHER NOTE** If students' algebra skills are weak, an extra day may need to be added to work on this handout in more depth. **STATE RESOURCES** **TEXTEAMS: High School Geometry: Supporting TEKS and TAKS:** I – Structure; 4.0 Conjectures, 4.1, Act. 1 (Triangles Tell It All) may be used to reinforce these concepts or used as alternate activities.

Instructional Procedures	Notes for Teacher
EVALUATE 1. Debrief handout: **Can You Justify It?** in whole-group discussion. 2. Distribute the handout: **Looking at It Logically** to each student. 3. Students should work the handout individually as an assessment.	**Suggested Day 4** **MATERIALS** • Handout: **Looking at It Logically** (1 per student) • protractor • ruler **TEACHER NOTE** This activity should be completed independently to assess student knowledge of the concepts taught in the lesson.

True/False and Logic



Have you ever wondered how a computer works? Today we take for granted the many programs that allow us to do everything from manage our money to communicate around the world by e-mail. In fact, many of the technology applications that we use daily were not even around twenty years ago! So what makes these technological wonders possible? Surprisingly, the technology that you and I enjoy today has its roots in the 1800's!

The basis for the modern computer, and anything that uses microprocessors for that matter, is something called *Boolean Logic* developed by George Boole in the mid 1800's. *Boolean Logic* is based on the premise that a particular statement can only have one of two values, *true* or *false*. The modern computer combines *Boolean Logic* (the TRUE or FALSE system) with the *Binary* number system (which only consists of *0* and *1*) and assigns pieces of data a code that consists of zeros and ones. The codes are assigned with the help of what is known as *logic gates* or *operators*. For example, the letter *A* is equivalent to the binary code *01000001*. Computers transmit and receive data in streams of binary codes that are then converted into representations that are meaningful to us.

A statement is TRUE, if it is TRUE for every case.

A statement is FALSE, if it is FALSE at least once.

True/False Quiz KEY

Consider the following statements and determine whether they are TRUE or FALSE. If your answer is false, give a counter-example proving it false.

1. $a(b + c) = ab + ac$ True
2. Birds fly. False (Penguins do not fly, but they are birds.)
3. Deer have antlers. False (Fawns and does are deer, but they do not have antlers.)
4. Texas is in North America. True
5. A rectangle has 4 right angles. True
6. Fish live in lakes and streams. False (Some fish live in the ocean.)
7. Athletes play football. False (Many athletes play other games such as baseball.)
8. Geometry is a mathematics course. True
9. Humans are insects. False (Humans are not insects.)
10. Austin, Texas is the capital of Texas. True

True/False Quiz

Consider the following statements and determine whether they are TRUE or FALSE. If your answer is false, give a counter-example proving it false.

1. $a(b + c) = ab + ac$

2. Birds fly.

3. Deer have antlers.

4. Texas is in North America.

5. A rectangle has 4 right angles.

6. Fish live in lakes and streams.

7. Athletes play football.

8. Geometry is a mathematics course.

9. Humans are insects.

10. Austin is the capital of Texas.

True/False and Logic Statements (pp. 1 of 3) KEY

True/False Summary

- A statement is true, if it is true for all cases. (No untrue case can be found.)
- A statement is false, if it is false at least once. (It does not matter if some cases are TRUE.)
- A counter-example is an example that contradicts a statement (shows statement is FALSE).
    - It only takes **one** *counterexample* to show that a statement is FALSE.
    - *Counterexamples* should be as specific as possible.
    - *Counterexamples* may be verbal, drawings, etc.
- Quantifiers such as *some*, *all*, or *none* can be used to change the truth value of a statement.

EXAMPLES: Determine if the following statement is true or false. If false, give a specific counterexample; then rewrite the statement to make it true.

1. All gasoline is sold by FINA. FALSE. Texaco sells gasoline. Some gasoline is sold by FINA.

2. The square of a number is always positive. FALSE. The square of zero is zero which is neither positive nor negative. The square of a number is sometimes positive.

Now that you have an understanding of TRUE and FALSE, consider the logic statements AND and OR, which couple two or more statements into one. AND and OR operators are used extensively in electronic applications.

A common nursery rhyme starts out, “Jack and Jill went up the hill…” As an AND statement, this is expanded to “Jack went up the hill AND Jill went up the hill.” For the given statement, “Jack went up the hill AND Jill went up the hill” there are four possible scenarios that could determine the truth value (TRUE/FALSE) of the statement. The scenarios are summarized in a table below. Use the information to determine if the statement is TRUE or FALSE.

AND Statement

Truth value of “Jack went up the hill.”	**Truth value of “Jill went up the hill.”**	**Truth Value of “Jack went up the hill AND Jill went up the hill.”**
True	True	TRUE
True	False (Jill did not go up the hill.)	FALSE
False (Jack did not go up the hill.)	True	FALSE
False (Jack did not go up the hill.)	False (Jill did not go up the hill.)	FALSE

3. Out of the four possible cases, which ones were TRUE? Why?
   The statement is TRUE, when both go up the hill.

4. Out of the four possible cases, which ones were FALSE? Why?
   The statement is FALSE when at least one of them do not go up the hill.

True/False and Logic Statements (pp. 2 of 3) KEY

5. Based on your findings in the table, consider "*p* AND *q*" where *p* and *q* are statements. Describe the conditions that make "*p* AND *q*" TRUE in terms of the truth values of *p* and *q*.
   p AND q is TRUE, if both p and q are TRUE.

6. Based on your findings in the table, consider "*p* AND *q*" where *p* and *q* are statements. Describe the conditions that make "*p* AND *q*" FALSE in terms of the truth values of *p* and *q*.
   p AND q is FALSE, if either p or q are FALSE or both are FALSE.

Now consider the statement as an OR statement. As an OR statement, it is expanded to, "Jack went up the hill OR Jill went up the hill." Again, there are four possible scenarios that could determine the truth value (TRUE/FALSE) of the statement. The scenarios are summarized in a table below. Use the information to determine if the statement is TRUE or FALSE.

OR Statement

Truth value for "Jack went up the hill."	Truth value for "Jill went up the hill."	Truth Value of "Jack went up the hill OR Jill went up the hill."
True	True	TRUE
True	False (Jill did not go up the hill.)	TRUE
False (Jack did not go up the hill.)	True	TRUE
False (Jack did not go up the hill.)	False (Jill did not go up the hill.)	FALSE

7. Out of the four possible cases, which ones were TRUE? Why?
   The statement is TRUE, if at least one of them go up the hill.

8. Out of the four possible cases, which ones were FALSE? Why?
   The statement is FALSE when neither go up the hill.

9. Based on your findings in the table, consider "*p* OR *q*" where *p* and *q* are statements. Describe the conditions that make "*p* OR *q*" TRUE in terms of the truth value of *p* and *q*.
   p OR q is TRUE at least one of the statements or both are TRUE.

10. Based on your findings in the table, consider "*p* OR *q*" where *p* and *q* are statements. Describe the conditions that make "*p* OR *q*" FALSE in terms of the truth value of *p* and *q*.
    p OR q is FALSE when neither of the statements are TRUE (both are FALSE).

True/False and Logic Statements (pp. 3 of 3) KEY

Logic Statement Summary

- An AND statement is true when both statements are true.
- An AND statement is false when at least one statement is false.
- An OR statement is true when one of the statements or *both* are true.
- An OR statement is false when neither statement is true.
- Truth tables can be used to summarize conditions that make AND and OR statements TRUE or FALSE.

AND Truth Table

p	q	p AND q
T	T	T
T	F	F
F	T	F
F	F	F

OR Truth Table

p	q	p OR q
T	T	T
T	F	T
F	T	T
F	F	F

True/False and Logic Statements (pp. 1 of 3)

True/False Summary

- A statement is __________, if it is true for all cases. (No untrue case can be found.)
- A statement is __________, if it is false at least once. (It does not matter if some cases are TRUE.)
- A _______________ is an example that contradicts a statement (shows statement is FALSE).
  - o It only takes **one** *counterexample* to show that a statement is FALSE.
  - o *Counterexamples* should be as specific as possible.
  - o *Counterexamples* may be verbal, drawings, etc.
- _______________ such as some, all, or none can be used to change the truth value of a statement.

EXAMPLES: Determine if the following statement is true or false. If false, give a specific counterexample; then rewrite the statement to make it true.

1. All gasoline is sold by FINA. FALSE.

2. The square of a number is always positive.

Now that you have an understanding of TRUE and FALSE, consider the logic statements AND and OR which couple two or more statements into one. AND and OR operators are used extensively in electronic applications.

A common nursery rhyme starts out, “Jack and Jill went up the hill…” As an AND statement, this is expanded to “Jack went up the hill AND Jill went up the hill.” For the given statement, “Jack went up the hill AND Jill went up the hill” there are four possible scenarios that could determine the truth value (TRUE/FALSE) of the statement. The scenarios are summarized in a table below. Use the information to determine if the statement is TRUE or FALSE.

AND Statement

Truth value of “Jack went up the hill.”	Truth value of “Jill went up the hill.”	Truth Value of “Jack went up the hill AND Jill went up the hill.”
True	True	
True	False (Jill did not go up the hill.)	
False (Jack did not go up the hill.)	True	
False (Jack did not go up the hill.)	False (Jill did not go up the hill.)	

3. Out of the four possible cases, which ones were TRUE? Why?

4. Out of the four possible cases, which ones were FALSE? Why?

True/False and Logic Statements (pp. 2 of 3)

5. Based on your findings in the table, consider "*p* AND *q*" where *p* and *q* are statements. Describe the conditions that make "*p* AND *q*" TRUE in terms of the truth values of *p* and *q*.

6. Based on your findings in the table, consider "*p* AND *q*" where *p* and *q* are statements. Describe the conditions that make "*p* AND *q*" FALSE in terms of the truth values of *p* and *q*.

Now consider the statement as an OR statement. As an OR statement, it is expanded to, "Jack went up the hill OR Jill went up the hill." Again, there are four possible scenarios that could determine the truth value (TRUE/FALSE) of the statement. The scenarios are summarized in a table below. Use the information to determine if the statement is TRUE or FALSE.

OR Statement

Truth value for "Jack went up the hill."	Truth value for "Jill went up the hill."	Truth Value of "Jack went up the hill OR Jill went up the hill."
True	True	
True	False (Jill did not go up the hill.)	
False (Jack did not go up the hill.)	True	
False (Jack did not go up the hill.)	False (Jill did not go up the hill.)	

7. Out of the four possible cases, which ones were TRUE? Why?

8. Out of the four possible cases, which ones were FALSE? Why?

9. Based on your findings in the table, consider "*p* OR *q*" where *p* and *q* are statements. Describe the conditions that make "*p* OR *q*" TRUE in terms of the truth value of *p* and *q*.

10. Based on your findings in the table, consider "*p* OR *q*" where *p* and *q* are statements. Describe the conditions that make "*p* OR *q*" FALSE in terms of the truth value of *p* and *q*.

True/False and Logic Statements (pp. 3 of 3)

Logic Statement Summary

- An ___________ statement is true when both statements are true.
- An ___________ statement is false when at least one statement is false.
- An ___________ statement is true when one of the statements or *both* are true.
- An ___________ statement is false when neither statement is true.
- ______________________ can be used to summarize conditions that make AND and OR statements TRUE or FALSE.

AND Truth Table

p	q	p AND q
T	T	
T	F	
F	T	
F	F	

OR Truth Table

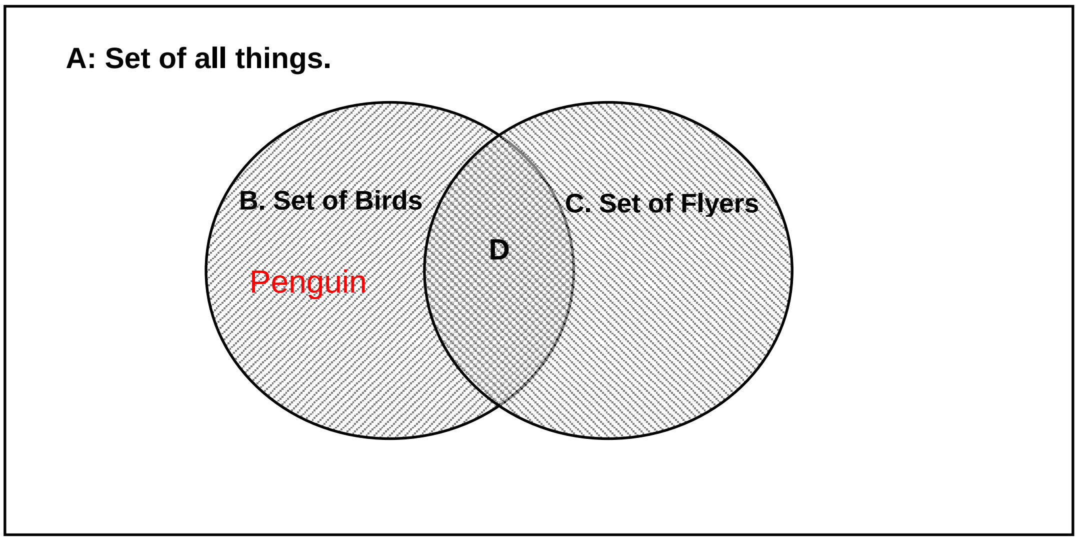
p	q	p OR q
T	T	
T	F	
F	T	
F	F	

Venn There! Done That! (pp. 1 of 4) KEY

Another way to illustrate logic statements and TRUE/FALSE is to use a graphical representation called a *Venn diagram*. *Venn diagrams* are used to show relationships between sets or groups of things.

Let's consider some of the statements from the TRUE/FALSE Quiz.

1. Consider the statement, "Birds fly," and the *Venn diagram* below. Respond to the questions that follow. Recall that this statement is FALSE because you were able to find a *counterexample* for it.



a. Notice that set B overlaps set C in a region labeled D. What types of things are in set B?
Set of all birds.

b. What types of things are in set C?
Set of all things that fly.

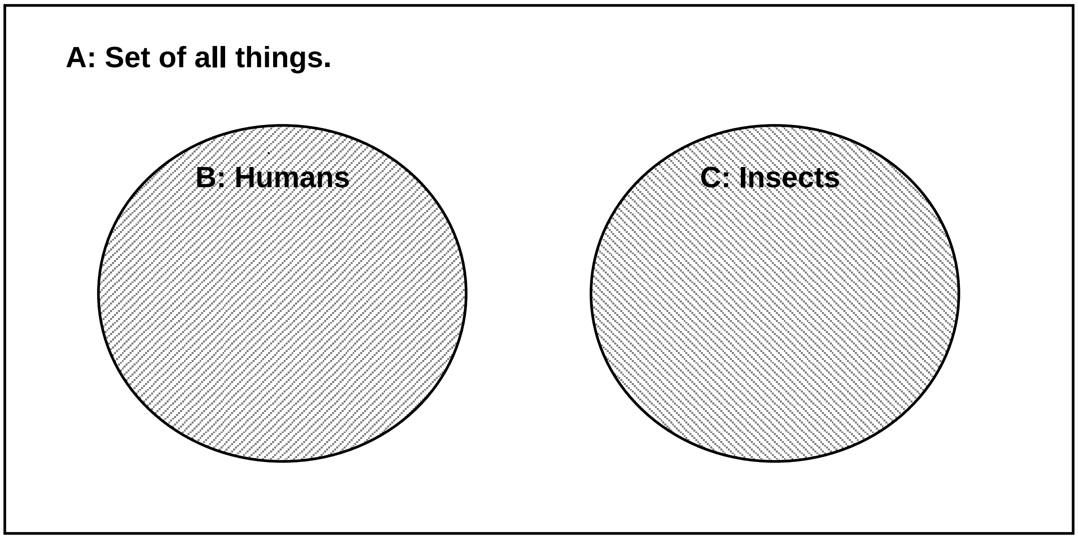
c. What types of things are in set D? What is the significance of set D in terms of the truth value of the statement?
Set of birds that fly. Things in set D make the statement TRUE.

d. Recall your counterexample for the statement, "Birds fly." Write your counterexample in the appropriate place on the Venn diagram to show that the statement, "Birds fly," is FALSE.
See diagram above.

e. Using the Venn diagram above write a TRUE statement using the word *some*.
Some birds fly.

Venn There! Done That! (pp. 2 of 4) KEY

2. Consider the statement, "Humans are insects," and the *Venn diagram* below. Respond to the questions that follow. Recall that this statement is FALSE because you were able to find a *counterexample* for it.



a. What was your counterexample for the statement, "Humans are insects"?
Myself. I'm not an insect.

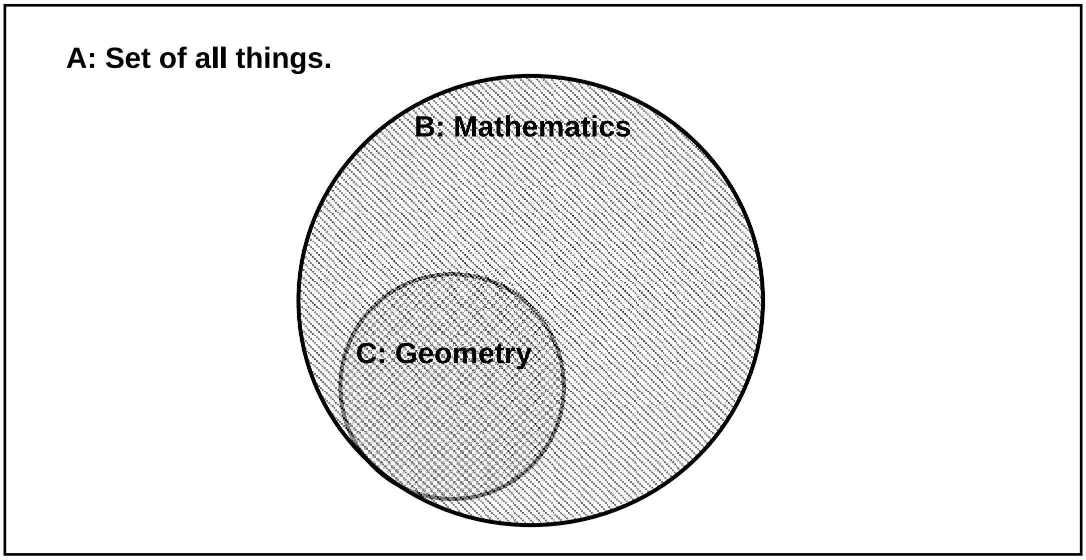
b. Notice in this case, counterexamples are numerous; I'm human and I'm not an insect, you're human and you're not an insect, etc. The two sets *B: Humans* and *C: Insects* are said to be *mutually exclusive*, meaning they have nothing in common. You will notice in the Venn diagram that they do not touch or overlap. Rewrite the statement, "Humans are insects," using the word *no* so that it is TRUE.
No humans are insects.

c. Using the Venn diagram write a TRUE statement.
No insects are human.

d. Using the Venn diagram write a FALSE statement that is different from the original.
Insects are human.

Venn There! Done That! (pp. 3 of 4) KEY

3. Consider the statement, "Geometry is a mathematics course," and the *Venn diagram* below. Respond to the questions that follow.



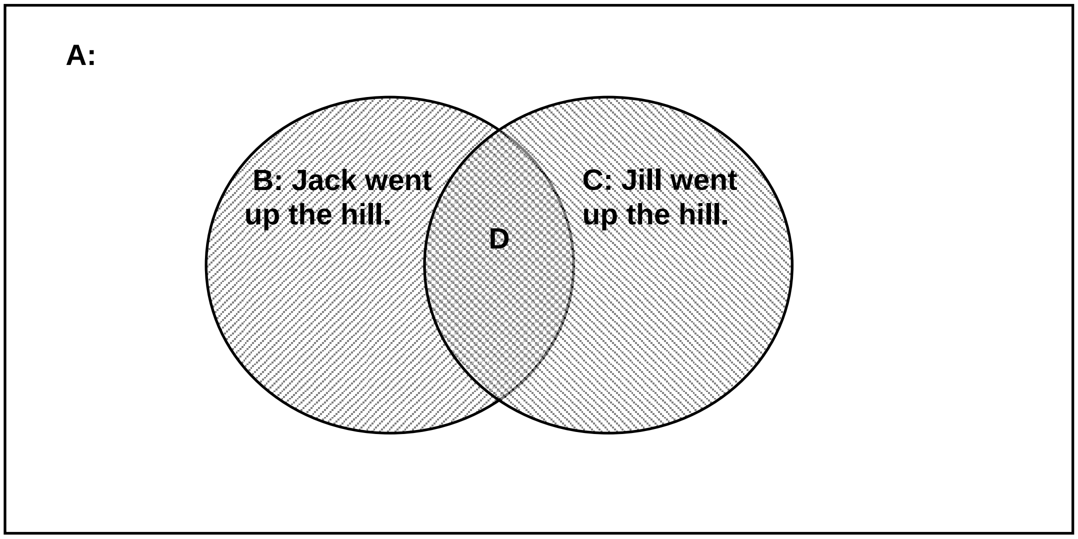
a. Recall that the statement, "Geometry is a mathematics course," is TRUE. Explain in terms of the sets in the Venn diagram why this is so.
Set C is contained in (subset of) set B. Therefore, geometry must be a mathematics course.

b. What types of things are in set B not including set C?
Math courses that are not geometry

c. Use the Venn diagram to write a statement that is FALSE.
Math courses are geometry course.

Venn There! Done That! (pp. 4 of 4) KEY

4. Consider the Venn diagram below that depicts the nursery rhyme, “Jack and Jill went up the hill…” Respond to the questions that follow.



a. Consider the statement, “Jack went up the hill AND Jill went up the hill.” Which part(s) of the Venn diagram depict when this statement is TRUE? Explain how you know in terms of the Venn diagram.
Region D (where sets B and C overlap) makes the statement TRUE. This region represents when both go up the hill.

b. Which part(s) of the Venn diagram depict when this statement is FALSE? Explain how you know in terms of the Venn diagram.
Set B excluding set C or set C excluding set B would make the statement FALSE. This represents when only one goes up the hill.

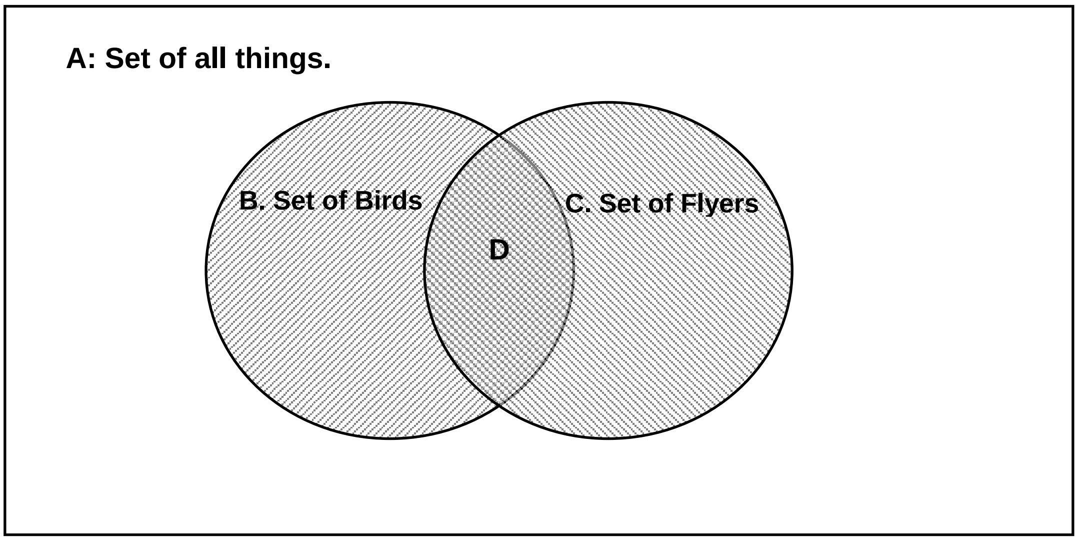
Set A makes the statement FALSE. This represents when neither go up the hill.

Venn There! Done That! (pp. 1 of 4)

Another way to illustrate logic statements and TRUE/FALSE is to use a graphical representation called a *Venn diagram*. *Venn diagrams* are used to show relationships between sets or groups of things.

Let's consider some of the statements from the TRUE/FALSE Quiz.

1. Consider the statement, "Birds fly," and the *Venn diagram* below. Respond to the questions that follow. Recall that this statement is FALSE because you were able to find a *counterexample* for it.



   a. Notice that set B overlaps set C in a region labeled D. What types of things are in set B?

   b. What types of things are in set C?

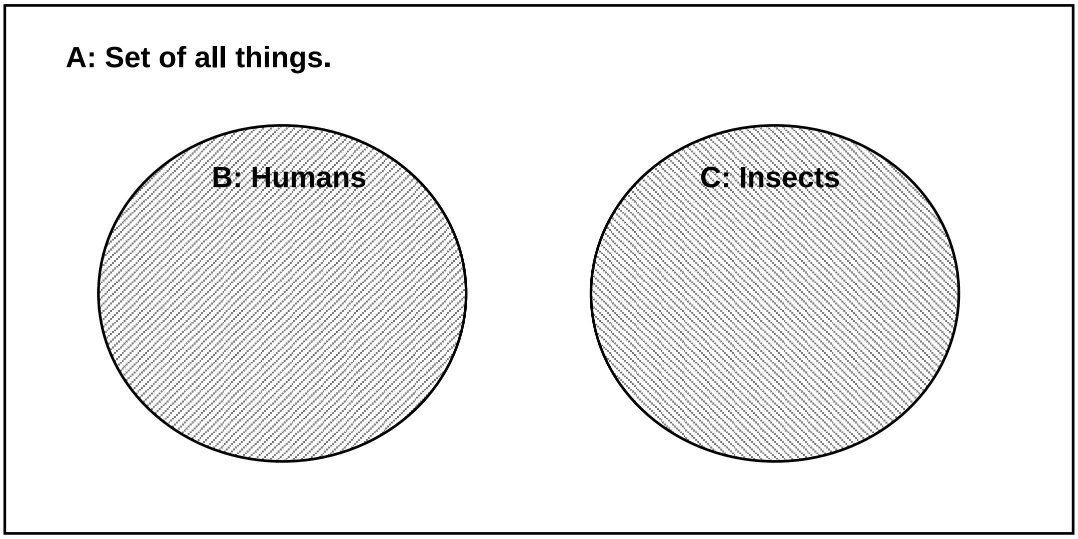
   c. What types of things are in set D? What is the significance of set D in terms of the truth value of the statement?

   d. Recall your counterexample for the statement, "Birds fly." Write your counterexample in the appropriate place on the Venn diagram to show that the statement, "Birds fly," is FALSE.

   e. Using the Venn diagram above write a TRUE statement using the word *some*.

Venn There! Done That! (pp. 2 of 4)

2. Consider the statement, “Humans are insects,” and the *Venn diagram* below. Respond to the questions that follow. Recall that this statement is FALSE because you were able to find a *counterexample* for it.



   a. What was your counterexample for the statement, “Humans are insects”?

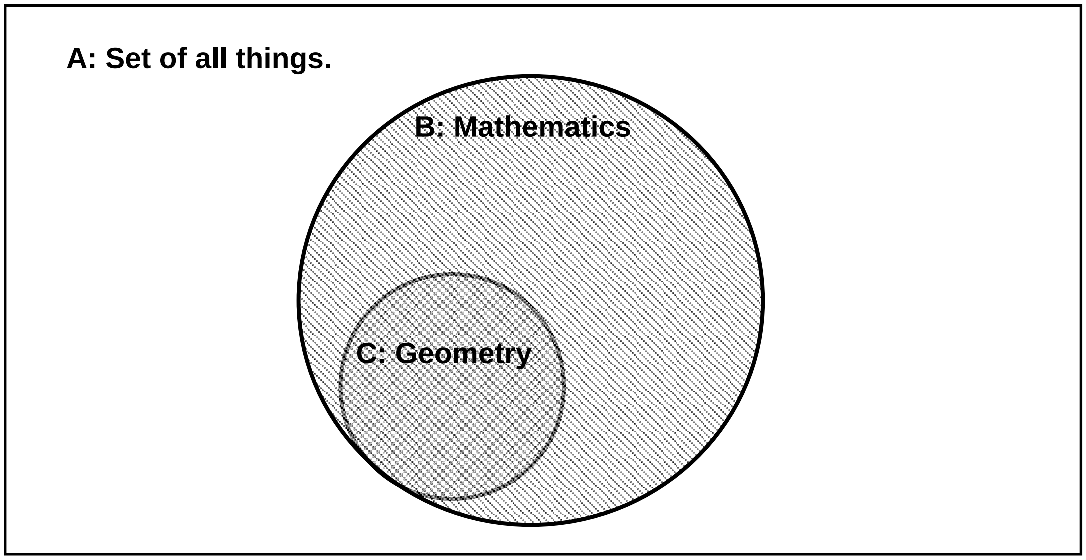
   b. Notice in this case, counterexamples are numerous; I’m human and I’m not an insect, you’re human and you’re not an insect, etc. The two sets *B: Humans* and *C: Insects* are said to be *mutually exclusive*, meaning they have nothing in common. You will notice in the Venn diagram that they do not touch or overlap. Rewrite the statement, “Humans are insects,” using the word *no* so that it is TRUE.

   c. Using the Venn diagram write a TRUE statement.

   d. Using the Venn diagram write a FALSE statement that is different from the original.

Venn There! Done That! (pp. 3 of 4)

3. Consider the statement, “Geometry is a mathematics course,” and the *Venn diagram* below. Respond to the questions that follow.



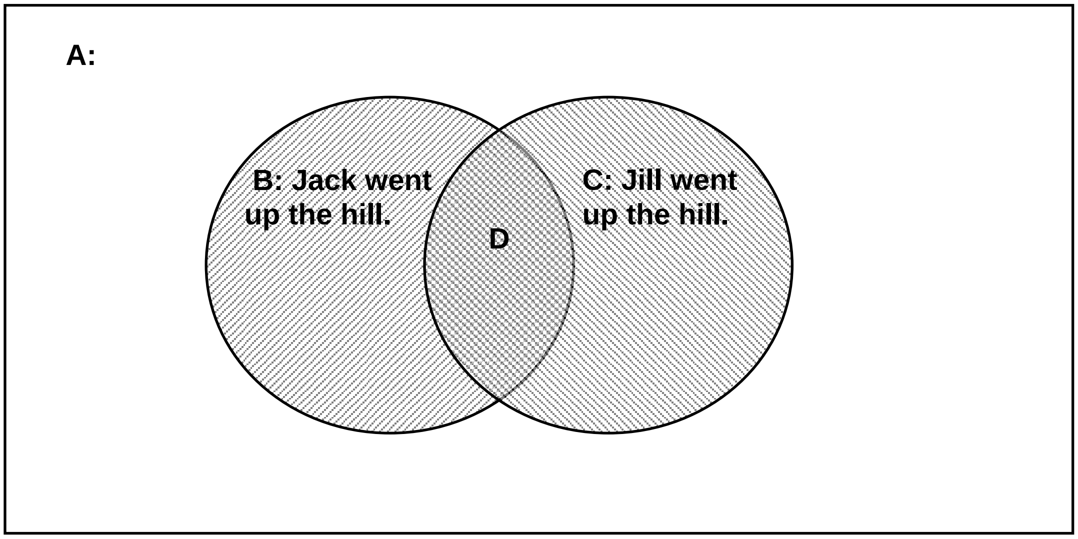
   a. Recall that the statement, “Geometry is a mathematics course,” is TRUE. Explain in terms of the sets in the Venn diagram why this is so.

   b. What types of things are in set B not including set C?

   c. Use the Venn diagram to write a statement that is FALSE.

Venn There! Done That! (pp. 4 of 4)

4. Consider the Venn diagram below that depicts the nursery rhyme, “Jack and Jill went up the hill…” Respond to the questions that follow.



a. Consider the statement, “Jack went up the hill AND Jill went up the hill.” Which part(s) of the Venn diagram depict when this statement is TRUE? Explain how you know in terms of the Venn diagram.

b. Which part(s) of the Venn diagram depict when this statement is FALSE? Explain how you know in terms of the Venn diagram.

Practicing True/False and Logic KEY

Use your previous knowledge to determine whether the following statements are TRUE or FALSE. If FALSE, give a counterexample.

1. If you are older than 12, then you are a teenager.
   FALSE, I'm 38, although my wife says I act like a teenager.

2. For all numbers *a* and *b*, $a + b = b + a$.
   TRUE

3. A square is a rectangle.
   TRUE

4. A rectangle is a square.
   FALSE, not all rectangles have 4 congruent sides.

5. If AM = MB then M is the midpoint of segment AB.
   FALSE, consider isosceles $\triangle AMB$ such that AM=MB.

6. If $x^2 = 25$ then $x = 5$.
   FALSE, x could be -5.

7. A math student is a geometry student.
   FALSE, the student could be an algebra student.

8. A geometry student is a math student.
   TRUE

Describe the conditions that make the statement TRUE and the conditions that make the statement FALSE.

9. James went to the movies **and** went to eat pizza.

   TRUE: James must go to movies. James must eat pizza.
   FALSE: James goes to movies, does not eat pizza.
   James does not go to movies, does eat pizza.
   James does not go to movies, does not eat pizza.

10. Cynthia will go to Six Flags **or** to the water park.

    TRUE: Cynthia goes to Six Flags; Cynthia goes to the water park.
    Cynthia goes to Six Flags; Cynthia does not go to the water park.
    Cynthia does not go to Six Flags; Cynthia goes to the water park.
    FALSE: Cynthia does not go to Six Flags; Cynthia does not go to the water park.

Practicing True/False and Logic

Use your previous knowledge to determine whether the following statements are TRUE or FALSE. If FALSE, give a counterexample.

1. If you are older than 12, then you are a teenager.
2. For all numbers *a* and *b*, $a + b = b + a$.
3. A square is a rectangle.
4. A rectangle is a square.
5. If AM = MB then M is the midpoint of segment AB.
6. If $x^2 = 25$ then $x = 5$.
7. A math student is a geometry student.
8. A geometry student is a math student.

Describe the conditions that make the statement TRUE and the conditions that make the statement FALSE.

9. James went to the movies **and** went to eat pizza.
10. Cynthia will go to Six Flags **or** to the water park.

On One Condition (pp. 1 of 3) KEY

Use the following conditional statement in determining your responses:

If I get paid today, then I will take you to the movies.

1. Notice the statement has two KEY parts, the *if* part or the *hypothesis*, and the *then* part or the *conclusion*. Write the conditional below. Underline *hypothesis* with one line (do not include the word *if*) and underline the *conclusion* with two lines (do not include the word *then*).
   Hypothesis: *I get paid today*
   Conclusion: *I will take you to the movies.*

2. Recall from the previous lesson that if a statement is TRUE, then its opposite is FALSE.
   a. Assuming the *hypothesis* of the above statement is TRUE, what condition would make the *hypothesis* FALSE?
      I do not get paid today.

   b. Assuming the *conclusion* of the above statement is TRUE, what condition would make the *conclusion* FALSE?
      I do not take you to the movies.

Conditional Statements Summary

- A conditional statement consists of a hypothesis and a conclusion.
- The hypothesis is the *if* part.
- The conclusion is the *then* part.
- A conditional is FALSE if the hypothesis is TRUE and the conclusion FALSE. Counter-example is any case that shows the conditional to be false. What would be the counter-example to the movies situation? I get paid today, but I do not take you to the movies.
- Given two statements *p* and *q*, "If *p*, then *q*," can be written as:
  - "*q* if *p*," or
  - "*p* implies *q*."

Conditional Truth Table for "If *p* then *q*."		
Hypothesis: *p*	**Conclusion: *q***	**If *p* then *q*.**
T	**T**	**T**
T	**F**	**F**
F	**T**	**T**
F	**F**	**T**

3. Consider the conditional statement, "If you live in the United States, then you live in Texas."
   a. What is the hypothesis?
      You live in the United States.

   b. What is the conclusion?
      You live in Texas.

On One Condition (pp. 2 of 3) KEY

c. Is the conditional TRUE or FALSE? How do you know? Provide a counterexample if FALSE.
   FALSE You could live in Arizona. (Arizona is in the U.S. but not in Texas.)

d. Rewrite the conditional in "q if p," form.
   You live in Texas if you live in the United States.

e. Rewrite the conditional in "p implies q," form.
   Living in the United States implies that you live in Texas.

Consider the statement, "*If you live in the United States, then you live in Texas.*"

4. Write a conditional statement using the phrases "*you live in Texas*" and "*you live in the United States*" that is similar to the one above but that is TRUE.
   If you live in Texas then you live in the United States.

5. How is your statement different from the original?
   The hypothesis and conclusion are different.

6. What did you change in terms of the hypothesis and conclusion? Explain.
   The hypothesis and conclusion have been switched.

When the hypothesis and conclusion are switched, the resulting conditional statement is called the ***converse*** of the original statement.

Go back to the original statement, "*If you live in the United States, then you live in Texas.*"

7. What is the negation of (opposite in meaning) the hypothesis?
   You do not live in the United States.

8. What is the negation of (opposite in meaning) the conclusion?
   You do not live in Texas.

9. Rewrite the original statement using the negation of the original hypothesis as the new hypothesis and the negation of the conclusion as the new conclusion.
   If you do not live in the United States, then you do not live in Texas.

When you negated both the hypothesis and the conclusion, the resulting conditional statement is called the ***inverse*** of the original statement. Is the inverse statement TRUE or FALSE? True

Go back to the original statement, "*If you live in the United States, then you live in Texas.*"

10. Rewrite the original statement using the negation of both the hypothesis and conclusion, and switching their positions.
    If you do not live in Texas, then you do not live in the United States.

When you negated both the hypothesis and the conclusion and switched their positions, the resulting conditional statement is called the ***contrapositive*** of the original statement. Is the inverse statement TRUE or FALSE? False

On One Condition (pp. 3 of 3) KEY

Refer back to the previous exercises. Summarize below how to construct the *converse*, *inverse*, and *contrapositive* given a conditional statement.

Converse:
Reverse the role of the hypothesis and conclusion.

Inverse:
Negate both the hypothesis and conclusion.

Contrapositive:
Reverse and negate both the hypothesis and conclusion.

Given the conditional, "*If you play football then you are an athlete."* Use the table below to find the related statements and the truth value of each.

	Statement	True/ False	Counterexample
Conditional	*If you play football, then you are an athlete.*	True	
Converse	If you are an athlete, then you play football.	False	Tennis, Golf, Baseball, etc.
Inverse	If you do not play football, then you are not an athlete.	False	Borg, Woods, Rodriguez, etc.
Contrapositive	If you are not an athlete, then you do not play football.	True	

What relationships are observed between the conditional statements and their True/False values?
Answers will vary. Sample:
The conditional and contrapositive are always the same, either both true or both false.
The converse and inverse are always the opposite of the conditional and contrapositive, and they are always the same.

On One Condition (pp. 1 of 3)

Use the following conditional statement in determining your responses:

If I get paid today, then I will take you to the movies.

1. Notice the statement has two KEY parts, the *if* part or the *hypothesis*, and the *then* part or the *conclusion*. Write the conditional below. Underline *hypothesis* with one line (do not include the word *if*) and underline the *conclusion* with two lines (do not include the word *then*).

2. Recall from the previous lesson that if a statement is TRUE, then its opposite is FALSE.
   a. Assuming the *hypothesis* of the above statement is TRUE, what condition would make the *hypothesis* FALSE?

   b. Assuming the *conclusion* of the above statement is TRUE, what condition would make the *conclusion* FALSE?

Conditional Statements Summary

- A conditional statement consists of a _______________ and a _______________.
- The hypothesis is the _______________.
- The conclusion is the _______________.
- A conditional is FALSE if the hypothesis is _______________ and the conclusion _______________. Counter-example is any case that shows the conditional to be false. What would be the counter-example to the movies situation?
- Given two statements *p* and *q*, "If *p*, then *q*," can be written as:
  - "*q* if *p*," or
  - "*p* implies *q*."

Conditional Truth Table for "If *p* then *q*."		
Hypothesis: *p*	**Conclusion: *q***	**If *p* then *q*.**
T	**T**	
T	**F**	
F	**T**	
F	**F**	

3. Consider the conditional statement, "If you live in the United States, then you live in Texas."
   a. What is the hypothesis?

   b. What is the conclusion?

On One Condition (pp. 2 of 3)

c. Is the conditional TRUE or FALSE? How do you know? Provide a counterexample if FALSE.

d. Rewrite the conditional in “q if p,” form.

e. Rewrite the conditional in “p implies q,” form.

Consider the statement, “*If you live in the United States, then you live in Texas.”*

4. Write a conditional statement using the phrases “*you live in Texas*” and “*you live in the United States*” that is similar to the one above but that is TRUE.

5. How is your statement different from the original?

6. What did you change in terms of the hypothesis and conclusion? Explain.

When the hypothesis and conclusion are switched, the resulting conditional statement is called the ***converse*** of the original statement.

Go back to the original statement, “*If you live in the United States, then you live in Texas.”*

7. What is the negation of (opposite in meaning) the hypothesis?

8. What is the negation of (opposite in meaning) the conclusion?

9. Rewrite the original statement using the negation of the original hypothesis as the new hypothesis and the negation of the conclusion as the new conclusion.

When you negated both the hypothesis and the conclusion, the resulting conditional statement is called the ***inverse*** of the original statement. Is the inverse statement TRUE or FALSE?

Go back to the original statement, “*If you live in the United States, then you live in Texas.”*

10. Rewrite the original statement using the negation of both the hypothesis and conclusion, and switching their positions.

When you negated both the hypothesis and the conclusion and switched their positions, the resulting conditional statement is called the ***contrapositive*** of the original statement. Is the inverse statement TRUE or FALSE?

On One Condition (pp. 3 of 3)

Refer back to the previous exercises. Summarize below how to construct the *converse*, *inverse*, and *contrapositive* given a conditional statement.

Converse:

Inverse:

Contrapositive:

Given the conditional, "*If you play football then you are an athlete."* Use the table below to find the related statements and the truth value of each.

	Statement	**True/ False**	**Counterexample**
Conditional	*If you play football, then you are an athlete.*	True	
Converse			
Inverse			
Contrapositive			

What relationships are observed between the conditional statements and their True/False values?

Conditionals in Mathematics (pp. 1 of 2) KEY

1. Recall from previous lessons the notion of a postulate. A postulate is a statement in geometry that we assume to be TRUE without any proof. Often postulates are stated as conditionals. Consider the *Linear Pair Postulate* from a previous lesson.

   Linear Pair Postulate-If two angles form a linear pair then the two angles are supplementary.

   a. What is the hypothesis?

      Two angles form a linear pair.

   b. What is the conclusion?

      The two angles are supplementary.

   c. Complete the table below to find the related statements and their truth values. Be sure to sketch or state a counterexample if FALSE.

	Statement	True/ False	Counterexample
Conditional	*If two angles form a linear pair, then the two angles are supplementary.*	True	
Converse	If two angles are supplementary, then they form a linear pair.	False	Sketch non-adjacent supplementary angles.
Inverse	If two angles do not form a linear pair, then the two angles are not supplementary.	False	Sketch non-adjacent supplementary angles.
Contrapositive	If two angles are not supplementary, then the two angles do not form a linear pair.	True	

Conditionals in Mathematics (pp. 2 of 2) KEY

2. Consider the *Vertical Angle Theorem* from a previous lesson. The *Vertical Angle Theorem* simply says, "*Vertical Angles are congruent*," which can be worded as a conditional as below.

   Vertical Angle Theorem-If two angles are vertical angles then the two angles are congruent.

   a. What is the hypothesis?

      Two angles are vertical angles.

   b. What is the conclusion?

      The two angles are congruent.

   c. Complete the table below to find the related statements and their truth values. Be sure to sketch or state a counterexample if FALSE.

	Statement	**True/ False**	**Counterexample**
Conditional	*If two angles are vertical angles, then the two angles are congruent.*	True	
Converse	If two angles are congruent, then the two angles are vertical angles.	False	Sketch congruent non-vertical angles.
Inverse	If two angles are not vertical angles, then the two angles are not congruent.	False	Sketch congruent non-vertical angles.
Contrapositive	If two angles are not congruent, then the two angles are not vertical angles.	True	

Conditionals in Mathematics (pp. 1 of 2)

1. Recall from previous lessons the notion of a postulate. A postulate is a statement in geometry that we assume to be TRUE without any proof. Often postulates are stated as conditionals. Consider the *Linear Pair Postulate* from a previous lesson.

   Linear Pair Postulate-If two angles form a linear pair then the two angles are supplementary.

   a. What is the hypothesis?

   b. What is the conclusion?

   c. Complete the table below to find the related statements and their truth values. Be sure to sketch or state a counterexample if FALSE.

	Statement	True/ False	Counterexample
Conditional	*If two angles form a linear pair, then the two angles are supplementary.*	True	
Converse			
Inverse			
Contrapositive			

Conditionals in Mathematics (pp. 2 of 2)

2. Consider the *Vertical Angle Theorem* from a previous lesson. The *Vertical Angle Theorem* simply says, "*Vertical Angles are congruent*," which can be worded as a conditional as below.

   Vertical Angle Theorem-If two angles are vertical angles, then the two angles are congruent.

   a. What is the hypothesis?

   b. What is the conclusion?

   c. Complete the table below to find the related statements and their truth values. Be sure to sketch or state a counterexample if FALSE.

	Statement	True/ False	Counterexample
Conditional	*If two angles are vertical angles, then the two angles are congruent.*	True	
Converse			
Inverse			
Contrapositive			

Modern Day Sherlock Holmes (pp. 1 of 2)

Perhaps you have heard of Sherlock Holmes, the fictional creation of Scottish author Sir Arthur Conan Doyle (Wikipedia, 2007). Sherlock Holmes first appeared in publication in 1887 as a London-based detective famous for his intellectual prowess, and renowned for his use of deductive reasoning and astute observation to solve difficult cases (Wikipedia, 2007). Without a doubt he is one of the most famous fictional detectives ever created impacting both literature and modern criminology.

While Sherlock Holmes is a fictitious character, the accounts of his reasoning ability are firmly rooted in logical reasoning and are just as relevant today in solving crimes as they were over a century ago. Modern day investigators use two types of logical reasoning to help solve crimes and build cases to prosecute criminals. *Inductive* and *deductive* reasoning have been used for well over a hundred years in the investigation of crimes including robbery, theft, fraud and burglary (Benny, 2007). According to Daniel J. Benny, a private investigator in Pennsylvania, *inductive* and *deductive* reasoning are used extensively in developing criminal profiles in crimes ranging from simple robbery to global terrorism (2007).

Links of interest:
http://www.bennypi.com/home.nxg
http://www.becca-online.org/images/Inductive_Reasoning_-_Benny.pdf

References:

Benny, D.J. (2007) *The Uses of Inductive and Deductive Reasoning In Investigations and Criminal Profiling,* Retrieved from http://www.becca-online.org/images/Inductive_Reasoning_-_Benny.pdf June 21, 2007.

Sherlock Holmes, Retrieved from http://en.wikipedia.org/wiki/Sherlock_Holmes June 21, 2007.

CASE: Convenience Store Caper

A local convenience store, *The Handy Stop* which is adjacent to the Overlook Apartments, fell victim to a robber recently. The crime happened in a flash at 11:23 p.m. and details of the crime and perpetrator are sketchy at best as the store clerk Debra Doolittle was doing what she does best which is very little. The robber made off with various pastries and a stick of beef jerky, and has therefore been dubbed, “The Snack Cake Bandit.”

Modern Day Sherlock Holmes (pp. 2 of 2)

Profile 1:
Sergeant Sleuth, a seasoned detective of many years, is very familiar with convenience store robberies and has investigated many throughout his career. Based on his interviews with previous criminals who were convicted of similar crimes, Sergeant Sleuth recalls that over 85% of convenience store robberies in this particular city were committed by white males between the ages of 18-21 and who lived with a relative. Based on his knowledge of previous cases involving convenience store robberies, he recalls that most occur between the hours of 11:00 p.m. and 3:00 a.m. at locations that are within walking distance of a housing complex.

Sergeant Sleuth submits a criminal profile of the perpetrator to aid in solving the crime. He reasons that the perpetrator of *The Handy Stop* robbery is a white male between the ages of 18-21, who lives with a relative in the Overlook Apartments. He reasons that the perpetrator is most likely a student or unemployed as the robbery took place at 11:23 p.m. He further reasons that the suspect is likely to strike again at *The 24-7*, a convenience store on the other side of the Overlook Apartments.

Profile 2:
Sergeant Sherlock, no slouch when it comes to solving crimes, had the opportunity to interview the store clerk Debra Doolittle the next day after the robbery. Upon interviewing Doolittle, Sherlock learned that the perpetrator had a dark complexion and a tattoo bearing the words, “4-Life” on his left arm. Doolittle also stated that the perpetrator wore heavy work boots that were covered in mud which she later had to clean up. She told Sherlock that embedded in the mud that she cleaned up were tiny pieces of seashells. She also related that the suspect fled in a white SUV. Since *The Handy Stop* is nowhere near the ocean, Sherlock reasons that the perpetrator could be employed by *Ocean Designs* some 15 minutes away which uses seashells in making its products. After researching *Ocean Designs*, he learns that the company has a work shift that ends at 11:00 p.m. and restricts employment to those over the age of 21 for liability reasons.

Sergeant Sherlock submits a criminal profile of the perpetrator to aid in solving the crime. Based on his interviews with Debra Doolittle and *Ocean Designs*, Sherlock reasons that the perpetrator drives a white SUV and is a non-white male over the age of 21 with a tattoo bearing the words, “4-Life.” He further reasons that it is likely that the perpetrator is employed by *Ocean Designs* and works the shift ending at 11:00 p.m.

Modern Day Sherlock Holmes Questions KEY

The two "criminal profiles" were developed using different types of reasoning but revolving around the same crime scenario. Study each "criminal profile" for similarities and differences. Keep the questions below in mind as you study each "criminal profile" to help you compare the two. After discussing the "criminal profiles" with those in your group, respond to the questions below. Be prepared to share your findings with others.

1. How are the two profiles similar? Give specific examples.
   Both profiles identify a male as the perpetrator.

2. How are the profiles different? Give specific examples.
   Profiles disagree on race, age, mode of transportation, employment, etc.

3. How was each profile developed?
   Profile 1 was based on the history of related crimes.
   Profile 2 was based on details that revolve around this specific crime.

4. What are the strengths of each profile?
   Because profile 1 is based on historical data, it is statistically strong; however, because profile 2 is based on the specifics of the case, it too is strong.

5. What are the weaknesses of each profile?
   Profile 1 is weak because it is largely unrelated to the specifics of the case.

6. Which profile do you believe is better? Why?
   Answers may vary. At this point (because we do not know the outcome of the case), both profiles are viable and in fact both types of profiles (inductive and deductive) are used to profile criminals in the real world. With that said, because some specifics of the case are known, most students will identify profile 2 as the stronger.

7. Which profile is most closely related to pattern recognition?
   Profile 1

8. Which profile is most closely related to the facts of the specific case?
   Profile 2

9. Which profile do you think is based on inductive reasoning?
   Profile 1

10. Which profile do you think is based on deductive reasoning?
    Profile 2

    At this point, students may not understand which type of reasoning is which. Inductive reasoning was first introduced in the Patterns unit. However, students may not recall the term inductive reasoning as that was not the emphasis of the Patterns unit.

Modern Day Sherlock Holmes Questions

The two "criminal profiles" were developed using different types of reasoning but revolving around the same crime scenario. Study each "criminal profile" for similarities and differences. Keep the questions below in mind as you study each "criminal profile" to help you compare the two. After discussing the "criminal profiles" with those in your group, respond to the questions below. Be prepared to share your findings with others.

1. How are the two profiles similar? Give specific examples.

2. How are the profiles different? Give specific examples.

3. How was each profile developed?

4. What are the strengths of each profile?

5. What are the weaknesses of each profile?

6. Which profile do you believe is better? Why?

7. Which profile is most closely related to pattern recognition?

8. Which profile is most closely related to the facts of the specific case?

9. Which profile do you think is based on inductive reasoning?

10. Which profile do you think is based on deductive reasoning?

Inductive and Deductive Reasoning KEY

Inductive and Deductive Reasoning Summary

Inductive Reasoning	Deductive Reasoning
▪ Based on observations ▪ Based on past experience ▪ Moves from specific observations to create a general rule ▪ Is useful in providing a conjecture or hypothesis ▪ Does not always lead to a valid conjecture	▪ Based on facts, definitions, postulates, properties, and theorems ▪ Moves from general observations to specific result ▪ Is useful in proving conjectures ▪ Always leads to a valid conclusion provided assumptions are TRUE

Discuss each of the following and determine if the method of reasoning is inductive or deductive.

1. You observe that for the last 5 of 6 weeks, the school cafeteria has served chicken on Thursdays. Since tomorrow is Thursday, you conclude that the cafeteria will serve chicken. Inductive
2. Since your lowest grade in History on any assignment or test is 92, you conclude that your average grade in History class is 92 or above. Deductive
3. One of your car's tires is inflated to 40 psi. Since the side of the tire rates the maximum tire pressure at 35 psi, you conclude that the tire is over inflated. Deductive
4. Bob the Big Bad Boxer has won his last 11 matches in less than 5 rounds. You conclude that he will win his next match in less than 5 rounds. Inductive
5. Fred and Amanda both solved the same algebra problem. Fred's answer is x=2.5 while Amanda's answer is x=5/2. You conclude that both are correct. Deductive

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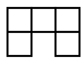
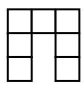
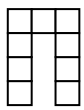
4. Bob the Big Bad Boxer has won his last 11 matches in less than 5 rounds. You conclude that he will win his next match in less than 5 rounds.

5. Fred and Amanda both solved the same algebra problem. Fred's answer is x=2.5 while Amanda's answer is x=5/2. You conclude that both are correct.

Can You Justify It? (pp. 1 of 4) KEY

INDUCTIVE REASONING:
In Unit 3: Patterns, you used inductive reasoning to formulate a rule to predict values. For example, given the following table,

Term	Visual	Verbal	Process	Numeric Value
1		Top row has 3, and 2 columns of 1 each	3+2(1)	5
2		Top row has 3, and 2 columns of 2 each	3+2(2)	7
3		Top row has 3, and 2 columns of 3 each	3+2(3)	9
4		Top row has 3, and 2 columns of 4 each	3+2(4)	11

and looking at a specific pattern, you were able to generalize a rule to predict the number of blocks for any term number *x* as follows…

x		Top row has 3, and 2 columns of x each	3+2(x)	3 + 2x

However, no justification for the conjecture of $3 + 2x$ to model the pattern is given other than the fact that it works for each entry in the table. Although inductive reasoning is a powerful tool for making predictions given a pattern, a *conjecture* based on inductive reasoning lacks proof and may not be TRUE in some cases.

DEDUCTIVE REASONING:
Deductive reasoning on the other hand, takes general information and arrives at a specific result. Deductive reasoning always arrives at a valid or TRUE conclusion because it is based on facts, definitions, postulates or other statements known to be true.

Consider the following from Algebra. Suppose you are given the equation $3(x + 2) = 14 – x$ and you are asked to solve for *x*. Can you justify each step of the process with a fact, definition, postulate or other statement known to be true? Try it!

1. Given $3(x + 2) = 14 – x$, suppose you transform the equation so that is becomes $3x + 6 = 14 – x$.
    a. Describe what you would do to achieve this transformation. Distributive the 3.
    b. What property from algebra justifies this transformation? Distributive prop.

Can You Justify It? (pp. 2 of 4) KEY

2. Suppose $3x + 6 = 14 – x$ is transformed to become $4x + 6 = 14$.
   a. Describe what you would do to achieve this transformation. Add x to both sides.
   b. What property from algebra justifies this transformation? Addition prop.

3. Suppose $4x + 6 = 14$ is transformed to become $4x = 8$.
   a. Describe what you would do to achieve this transformation. Add -6 to both sides.
   b. What property from algebra justifies this transformation? Addition prop.

4. Suppose $4x = 8$ is transformed to become $x = 2$.
   a. Describe what you would do to achieve this transformation. Multiply both sides by ¼.
   b. What property from algebra justifies this transformation? Multiplication prop.

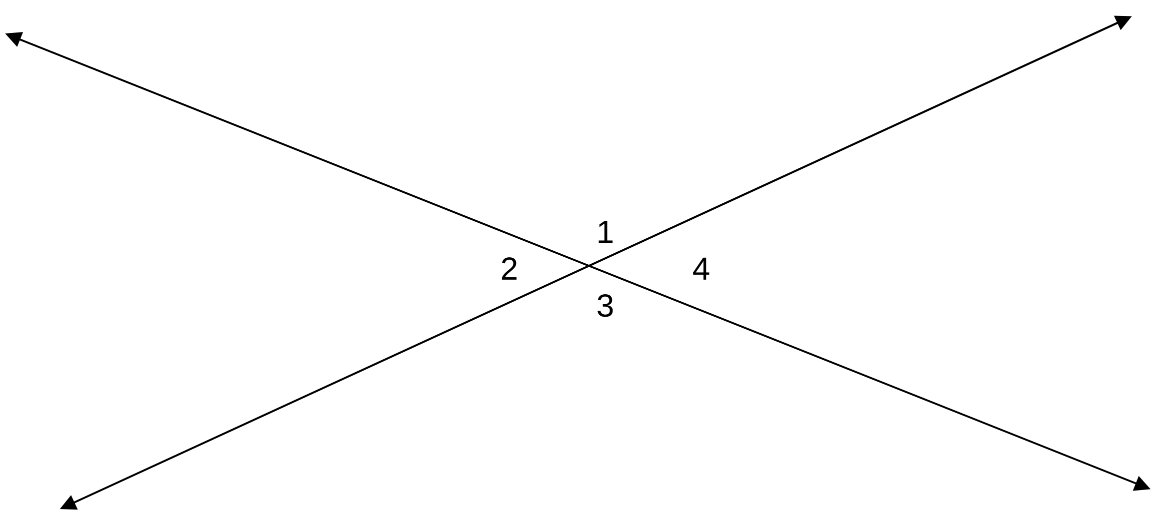
Solving the above equation is an example of a deductive thought process. Deductive reasoning is useful in proving arguments in mathematics, especially geometry. Sometimes it is useful to arrange deductive processes in a table format.

5. Below is a table of *Statements* and *Justifications* from the example above. The *Statements* are provided for you. Fill in the *Justification* column with the appropriate reason based on your responses above.

Statement	Justification
$3(x + 2) = 14 – x$	Given
$3x + 6 = 14 – x$	Distributive property
$4x + 6 = 14$	Addition property
$4x = 8$	Addition property
$x= 2$	Multiplication property

The table format provides a useful organizer for what we call a *proof* in geometry. In Unit 1: Angles, you conjectured about certain angle relationships. Let's revisit some of those relationships.

6. Using the figure below and a protractor, measure each of the following angles. What appears to be true for angles that are across from each other (Vertical angles)?



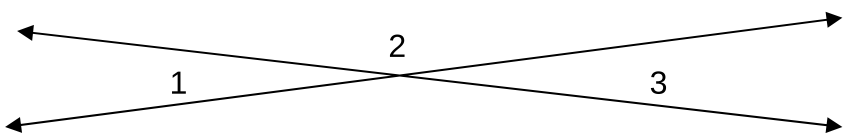
Can You Justify It? (pp. 3 of 4) KEY

7. Using a straight edge, draw two other examples of vertical angles formed by intersecting lines. Measure the angles formed, or use paper folding techniques to compare the angle measures of vertical angle pairs. What appears to be true about the vertical angle pairs in you drawings?
   They appear to be congruent.

8. Based on your observations and responses to questions 6 and 7, use inductive reasoning to form a conjecture about vertical angle pairs. Record you conjecture below.
   Vertical angles are congruent.

Perhaps you remember form Unit 1: Angles, the theorem *Vertical angles are congruent.* Was this your conjecture in question 8? Your conjecture about vertical angles above is only based on three observations. Although that may be enough to convince you that the statement *Vertical angles are congruent* is TRUE, it does not provide a reliable deductive argument. Let's see if we can model a deductive argument for *Vertical angles are congruent*.

Consider the picture below for the following questions.



9. To get started with a proof or a deductive argument, two critical pieces need to be identified. First of all, we need to know what we are trying to prove in terms of the picture. In our case, we are trying to prove the statement *Vertical angles are congruent*. So, in terms of the picture we are trying to prove that $\angle 1 \cong \angle 3$. Secondly, we need to determine any relationships that can be justified by facts, definitions, postulates, or properties. What can you observe about the other angles in the picture?

10. Observe that $\angle 1$ and $\angle 2$ form a Linear Pair as do $\angle 2$ and $\angle 3$. What does it mean if two angles form a Linear Pair? How do you know?
    The angles that form a linear pair are supplementary. Linear Pair Postulate.

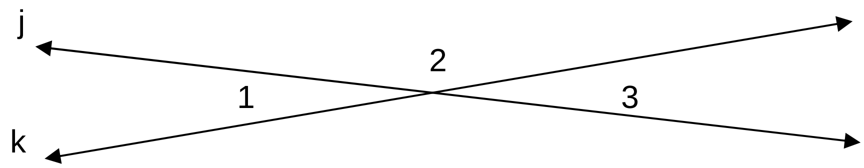
11. What does it mean for $\angle 1$ and $\angle 2$ to be supplementary? What does it mean for $\angle 2$ and $\angle 3$ to be supplementary? How do you know?
    The measures of the angles add up to 180. Definition of supplementary angles.

12. If $m\angle 1 + m\angle 2 = 180$ and $m\angle 2 + m\angle 3 = 180$, what must be true about the sums $m\angle 1 + m\angle 2$ and $m\angle 2 + m\angle 3$? How do you know?
    The sums are equal. This can be shown with substitution.

13. If $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$, what can you conclude? How do you know?
    $\angle 1 \cong \angle 3$ Add the opposite of $m\angle 2$ to both sides.

Can You Justify It? (pp. 4 of 4) KEY

14. Based on your answers above, complete the proof of the theorem- *Vertical angles are congruent*- in the table format below.



GIVEN: lines j and k intersect. $\angle 1$ and $\angle 3$ are vertical angles. $\angle 1$ and $\angle 2$ form a linear pair. $\angle 2$ and $\angle 3$ form a linear pair.

PROVE: $\angle 1 \cong \angle 3$

Statement	Justification
$\angle 1$ and $\angle 2$ form a linear pair. $\angle 2$ and $\angle 3$ form a linear pair.	Given
$\angle 1$ and $\angle 2$ are supplementary. $\angle 2$ and $\angle 3$ are supplementary.	Linear Pair Postulate – If two angles form a linear pair, then they are supplementary.
$m\angle 1 + m\angle 2 = 180$ and $m\angle 2 + m\angle 3 = 180$	Definition of supplementary angles – If two angles are supplementary, then their measures add up to 180.
$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	Substitution
$m\angle 1 = m\angle 3$	Addition property of equality – add $-(m\angle 2)$ to both sides.
$\angle 1 \cong \angle 3$	Definition of congruent angles – If two angles have the same measure, then they are congruent.

Can You Justify It? (pp. 1 of 4)

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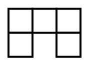
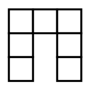
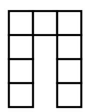
1		Top row has 3, and 2 columns of 1 each	3+2(1)	5
2		Top row has 3, and 2 columns of 2 each		
3				
4				

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x				

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Consider the following from Algebra. Suppose you are given the equation $3(x + 2) = 14 – x$ and you are asked to solve for *x*. Can you justify each step of the process with a fact, definition, postulate or other statement known to be true? Try it!

1. Given $3(x + 2) = 14 – x$, transform the equation so that is becomes $3x + 6 = 14 – x$.
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Can You Justify It? (pp. 2 of 4)

2. Suppose $3x + 6 = 14 - x$ is transformed to become $4x + 6 = 14$.
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4. Suppose $4x = 8$ is transformed to become $x = 2$.
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Solving the above equation is an example of a deductive thought process. Deductive reasoning is useful in proving arguments in mathematics, especially geometry. Sometimes it is useful to arrange deductive processes in a table format.

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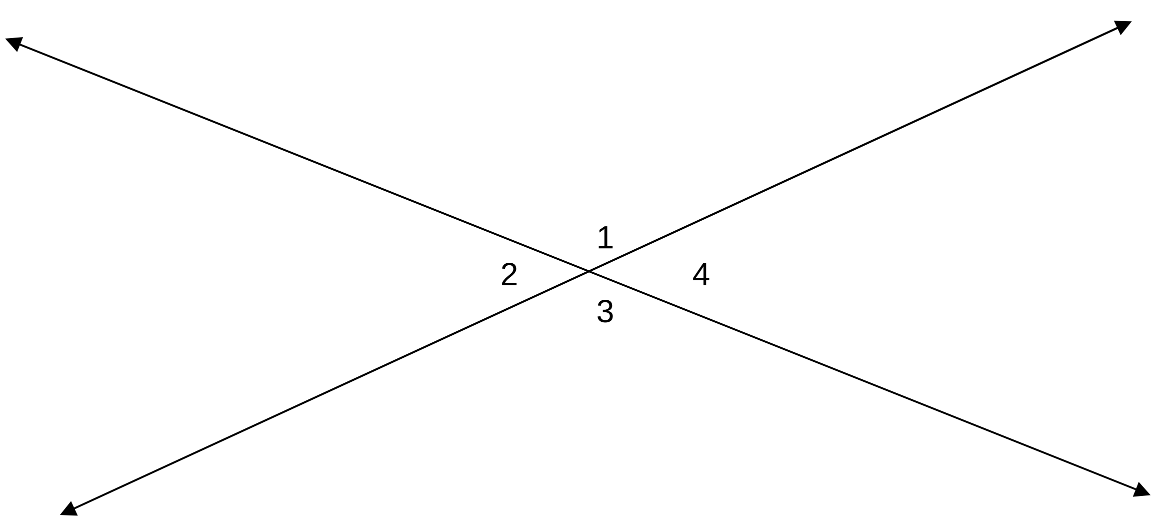

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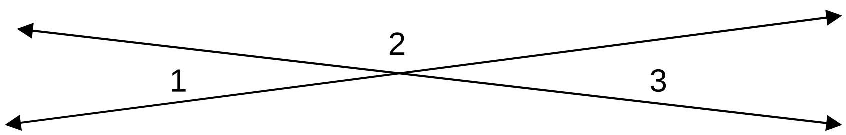

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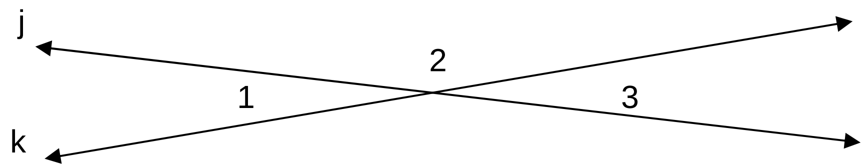
13. If $m\angle 1 + m\angle 2$ = $m\angle 2 + m\angle 3$, what can you conclude? How do you know?

Can You Justify It?

14. Based on your answers above complete the proof of the theorem: *Vertical angles are congruent* in the table format below.



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PROVE: $\angle 1 \cong \angle 3$

Statement	Justification
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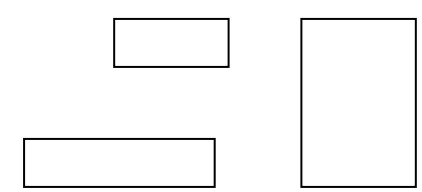
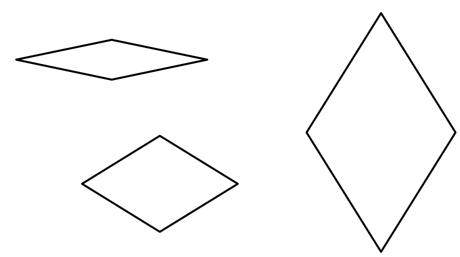
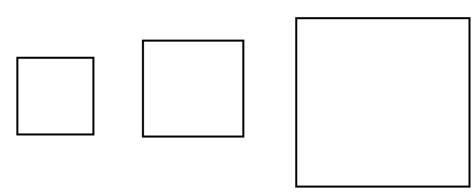
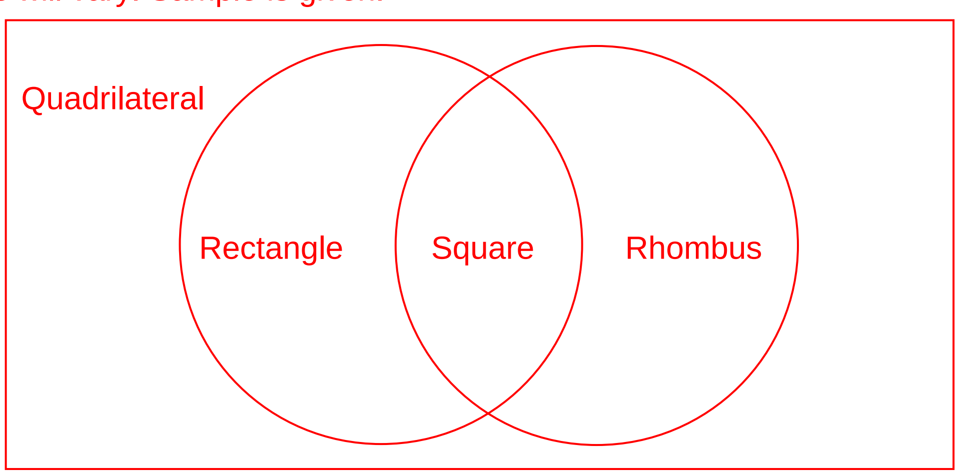
$\angle 1$ and $\angle 2$ are supplementary. $\angle 2$ and $\angle 3$ are supplementary.	
$m\angle 1 + m\angle 2 = 180$ and $m\angle 2 + m\angle 3 = 180$	
$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	
$m\angle 1 = m\angle 3$	
$\angle 1 \cong \angle 3$	

Looking at It Logically (pp. 1 of 3) KEY

In previous courses in mathematics, you have studied four sided polygons, including the rectangle, rhombus, and square. The table below summarizes critical attributes for each of the three four-sided polygons mentioned.

Type of Quadrilateral	Critical Attributes	Pictorial Examples
Rectangle	4 right angles	
Rhombus	4 congruent sides	
Square	4 right angles 4 congruent sides	

1. Using the information in the table, create a Venn diagram that depicts the relationship between the three types of quadrilaterals listed.
   Answers will vary. Sample is given.



Looking at It Logically (pp. 2 of 3) KEY

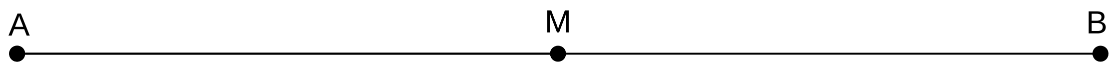
2. Use your Venn diagram to write a conditional statement that is TRUE.
   Answers will vary. Sample: If a figure is a square then it is a rhombus.

3. Use the table below to find the related statements and their truth values. Be sure to state or draw a counterexample using your Venn diagram if FALSE.
   Answers will vary. Sample is given based on sample answer in question 2.

	Statement	True/ False	Counterexample
Conditional	If a figure is a square, then it is a rhombus.	True	
Converse	If a figure is a rhombus, then it is a square.	False	
Inverse	If a figure is not a square, then it is not a rhombus.	False	
Contrapositive	If a figure is not a rhombus, then it is not a square.	True	

Looking at It Logically (pp. 3 of 3) KEY

When studying line segments, you discovered the Midpoint Theorem.



Midpoint Theorem: If M is the midpoint of $\overline{AB}$, then AM=1/2(AB) and MB=1/2(AB).

4. Construct an inductive argument for the Midpoint Theorem by creating examples that illustrate the theorem. Explain how your examples support an inductive argument for the Midpoint Theorem.
   Student products should support the midpoint theorem by showing length measures, which are half of the whole.

5. Using your previous knowledge of line segments, construct a deductive argument for the Midpoint Theorem using the two-column outline below.

   From the picture above:

   GIVEN: M is the midpoint of $\overline{AB}$

   PROVE: AM = $\frac{1}{2}$AB

Statement	**Justification**
M is the midpoint of $\overline{AB}$	Given
AM + MB = AB	Segment Addition Postulate
AM = MB	Definition of midpoint
AM + AM = AB	Substitution
2AM = AB	Collect like terms
AM = $\frac{1}{2}$AB	Multiplication property

Looking at It Logically **(pp. 1 of 3)**

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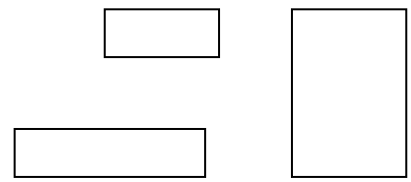
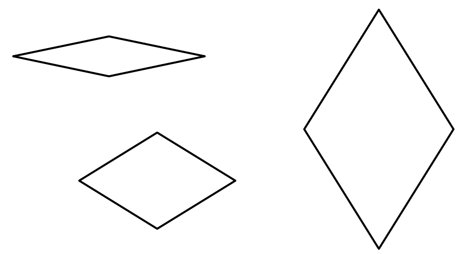
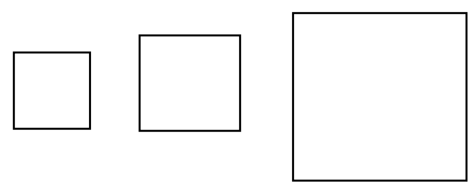
Rectangle	4 right angles	
Rhombus	4 congruent sides	
Square	4 right angles 4 congruent sides	

1. Using the information in the table, create a Venn diagram that depicts the relationship between the three types of quadrilaterals listed.

Looking at It Logically (pp. 2 of 3)

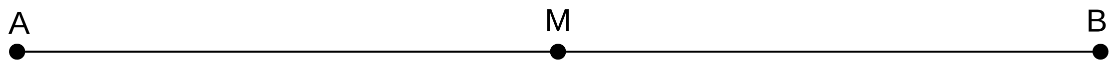
2. Use your Venn diagram to write a conditional statement that is TRUE.

3. Use the table below to find the related statements and their truth values. Be sure to state or draw a counterexample using your Venn diagram if FALSE.

	Statement	True/ False	Counterexample
Conditional		True	
Converse			
Inverse			
Contrapositive			

Looking at It Logically (pp. 3 of 3)

When studying line segments, you discovered the Midpoint Theorem.



Midpoint Theorem: If M is the midpoint of $\overline{AB}$, then AM=1/2(AB) and MB=1/2(AB).

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5. Using your previous knowledge of line segments, construct a deductive argument for the Midpoint Theorem using the two-column outline below.

   From the picture above:

   GIVEN: M is the midpoint of $\overline{AB}$

   PROVE: $AM = \frac{1}{2}AB$

Statement	**Justification**
M is the midpoint of $\overline{AB}$	
AM + MB = AB	
AM = MB	
AM + AM = AB	
2AM = AB	
$AM = \frac{1}{2}AB$	